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Dynamics of two-dimensional colloids on a disordered substrate

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Using Langevin simulations, we numerically study the dynamics of two-dimensional colloids on a disordered substrate. With a decreasing strength of the interaction between colloids, we find a crossover from elastic to plastic depinnings, where a substantial increase in the depinning force is observed. Furthermore, we find a dynamical phase transition from the moving liquid to the moving smectic at high driving forces by decreasing the temperature. Peak effect occurs in the dynamical critical driving force across the transition, accompanied by a clear crossing of velocity-force dependence curves.

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I. INTRODUCTION

Colloids display intriguing transitions between gas, liquid, solid and liquid crystalline phases. Recently, much attention has been focused on colloidal crystals, due to that they provide ideal model systems for investigating various problems in material science [1], physical chemistry [2], and condensed matter physics [3]. The structural and dynamical properties of two-dimensional (2D) clusters have motivated more interest and have been the subject of recent experimental studies [4] and numerical simulations [5]. In particular, the general problems of ordering and dynamics in 2D colloids have been addressed extensively [6–10].

Although the equilibrium thermodynamics of 2D colloidal crystals has been studied extensively, the nonequilibrium dynamics of 2D colloids on disordered substrates remains less explored. When brought into nonequilibrium, physical systems may exhibit many different kinds of pattern formation [11,12], which are much richer than the traditional phase transitions in equilibrium systems. Such exotic systems also include charge-density waves [13,14], vortices in superconductors [15], and Wigner crystals [16].

Recently, Reichhardt and Olson investigated the depinning dynamics of 2D colloids on a disordered substrate. They found a crossover from elastic to plastic depinnings with an increasing strength of the substrate [17]. However, the particle-particle interaction dependence of dynamics is still open. Dynamical properties of driven 2D systems interacting with quenched disorder are crucially affected by the interactions between particles. One intriguing example is a type-II superconductor in the mixed state where vortices are arranged in a vortex lattice. The competition between vortex-vortex and vortex-pin interactions can induce a threshold behavior. In addition, changes of the vortex interaction (by changing the external magnetic field) may lead to peak effect in the critical current density. It was believed that the softening of shear modulus of a vortex lattice and the easy compliance of the lattice with the pinning configuration result in a sudden increase of the critical current density before its disappearance [18]. The peak effect is associated with a disorder to order transition, as demonstrated by recent small angle neutron scattering experiments [19]. A natural and important question is whether these dynamical behaviors can occur in other systems which have different forms of particle-particle interactions, such as colloids, sliding charge-density waves, and driven electron crystals in the presence of random impurities.

In type-II superconductors, increasing temperatures transform the vortex glass phase into a vortex liquid [20]. An actual dynamical phase transition between the plastically deformed phase and a moving crystal for the vortex lattice was first proposed by Koshelev and Vinokur [21], whose idea is that the random pinning noise diminishes with increasing the vortex velocity, allowing a high-velocity reordering. For colloids, the temperature dependence needs further study. With the fixed pinning strength and particle-particle interaction, the temperature characterizes properties of the system mostly. It is significant and interesting to investigate the influence of temperature on the dynamical properties.

Motivated by the recent advance in colloidal experiments and simulations, we numerically investigate the dynamical properties of colloids. In this paper, influence of the temperature and the interaction between colloids on the dynamical behavior of 2D colloids was systematically considered for the first time, to the best of our knowledge. We will see that the system exhibits a series of intriguing phenomena.

II. MODEL

Numerical simulations offer a unique window through which one may view the qualitative behavior of colloids. The strength of colloid-colloid interaction and the strength of disorder can be precisely tuned. These are difficult to achieve in experiments. We investigate the dynamics of 2D colloids on a disordered substrate. The colloids are simulated by Langevin dynamics, and the motion of colloid is described by the equation

$$\frac{dR_i}{dt} = -\sum_{i\neq j} \nabla_i U_{c\text{-}c}(R_i - R_j) - \sum_{j'} \nabla_i U_{c\text{-}p}(R_i - r_{j'}) + F_i^L(t) + F_d, \qquad (1)$$

where $R_{i,j}$ and $r_{j'}$ are the colloid and pin coordinates.

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$U_{c\text{-}c}(R_i-R_j)$ and $U_{c\text{-}p}(R_i-r_{j'})$ are the colloid-colloid interaction potential and the colloid-pin interaction potential, respectively. F_d is an external applied drive, which may be induced by an external field (such as electric field) [3]. The Langevin force $F_i^L(t)$ describes the coupling with a heat bath and can be expressed by the correlation function [22]

$$\langle F_{i\alpha}^L(t)F_{j\beta}^L(t')> = 2\ \eta T\delta_{ij}\delta_{\alpha\beta}\delta(t-t'), \tag{2}$$

where subscripts α and β take values 1, 2, or 3, representing the Cartesian components of $F_i^L(t)$.

The colloids interact through a screened Coulomb potential [9,10,17]

$$U_{c\text{-}c}(R_i-R_j) = \frac{Q^2}{|R_i-R_j|}\exp(-\kappa|R_i-R_j|), \tag{3}$$

where Q the charge of colloidal particles, and $1/\kappa$ the screening length. By tuning the value of Q^2, we will change the interaction strength between colloids. We define $Q^2=\Gamma$.

Different from Ref. [17], where the quenched disorder is modeled as randomly placed parabolic traps, we choose the colloid-pin interaction as a conventional attracting Gaussian potential [22,23]

$$U_{c\text{-}p}(R_i-r_{j'}) = -f_{\mathrm{p}}\exp\left(-\frac{|R_i-r_{j'}|^2}{r_p^2}\right), \tag{4}$$

where f_{p} is a constant. We choose $f_{\mathrm{p}}=0.1$ in this paper. This potential can be produced by holographic optical tweezers technique in recent experiments [24].

All the lengths will be measured with respect to the lattice constant a_0 of the ideal triangular lattice. We take $\kappa=2/a_0$ and the scale for temperature is chosen as the "bare" Kosterlitz-Thouless melting temperature T_{m^0} [25]. The size of pinning centers, r_p, is chosen to be $r_{\mathrm{p}}=0.2\ a_0$. Colloids are initially placed in a perfect triangular lattice subject to periodic triangular boundary conditions. Pointlike pinning centers are randomly distributed on the substrate. The driving force F_{d} is increased from zero by a small increment along the horizontal symmetry axis (x axis) and the average colloid velocity $V_x=(1/N_c)\Sigma_{i=1}^{N_c}\mathbf{V}_i\cdot\hat{x}$ is measured at each increase. A time integration step of $\Delta t=0.001$ is used and averages are evaluated during 1×10^6 steps after equilibrium. We choose the size of system so large that the dynamical properties change little with varying the system size. So, the size influence can be ignored in our simulation.

III. NUMERICAL RESULTS AND DISCUSSION

In Fig. 1 we present series of velocity-force dependence (VFD) curves for different values of the strength of colloid-colloid interaction at a fixed temperature ($T/T_{\mathrm{m}^0}=0.1$). One can see that there exists a critical driving force (equals to the depinning force F_{c}) in each curve of VFD, below which the colloids are pinned and the velocities are generated mainly by rearrangements of topological defects, producing so small advances that can be neglected. We find a dramatic change in the gross feature of VFD curves as Γ is varied. For strong colloid-colloid interactions with large values of $\Gamma(\Gamma\geqslant6.0)$, the VFD response is basically linear above F_{c}. In this case, the colloids depin elastically and the motion of colloids is homogeneous above F_{c}. Each colloid keeps the same neighbors as it moves. The trajectories are shown in the inset (a) of Fig. 1, where we find a set of periodic elastic channels parallel to the driving force, consistent with recent simulations [17]. For weak colloid-colloid interactions with small values of $\Gamma(\Gamma\leqslant5.0)$, plastic flow takes place above F_{c}, and the VFD curves show a pronounced concave upward curvature, which is induced by defects in the flowing colloids. This is the typical characteristic of plastic depinning, as found in vortex lattice of superconductors [22]. The flow pattern is shown in the inset (b) of Fig. 1, where a fraction of colloids are found to move in the intricate network of channels. Different from the elastic case, these channels are not static but changeable over time. The flow consists of two components: unpinned individual colloids in the flow state and pinned colloids. The latter contributes to the flow through intermittent motion. In this case, the colloid medium is broken up by an external force and the transport is highly inhomogeneous.

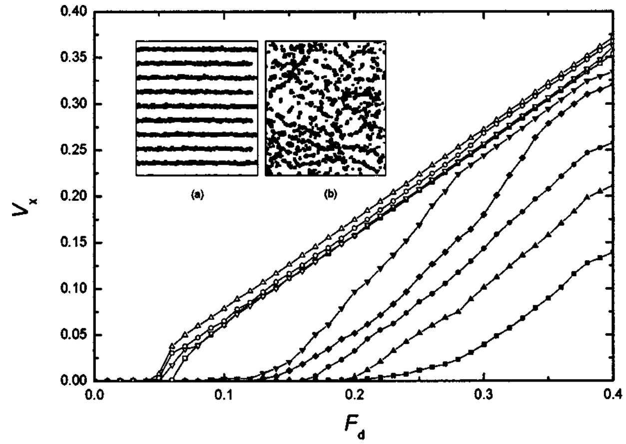

FIG. 1. Series of VFD curves for different values of Γ (from left to right) = 9, 8, 7, 6, 5, 4, 3, 2, and 1. Inset: flow pattern (trajectories) of colloids above depinning ($F_{\mathrm{d}}/F_{\mathrm{c}}=1.1$) for (a) $\Gamma=8$ and (b) $\Gamma=1$.

To identify the elastic to plastic crossover further, we examine the structure factor $S(k)=\langle|(1/N_C)\Sigma_i\exp[ik\cdot r_i(t)]|^2\rangle$ near depinning. For strong colloid-colloid interactions with large values of Γ, the interaction dominates. The lattice is stiff and the hexagonal order is robust. One can see this point from the sixfold coordinated Bragg peaks of $S(k)$ in Fig. 2(a). In this case, the tearing and defect formation of moving lattice may be ignored. Decreasing the strength of colloid-colloid interaction, disorder increases and plays a dominant role when Γ is below 6.0. The lattice is torn and sixfold coordinated Bragg peaks in $S(k)$ disappear, as shown in Fig. 2(b), where only a peak is found at the center ($K_x=K_y=0$). Figure 2(c) shows the average intensity of sixfold coordinated Bragg peaks $S(G)$, which is found to decrease with decreasing Γ. A substantial decrease in $S(G)$ is observed within the crossover from elastic to plastic depinnings.

In addition, we can see from Fig. 1 that F_{c} decreases with increasing values of Γ. This is consistent with recent simu-

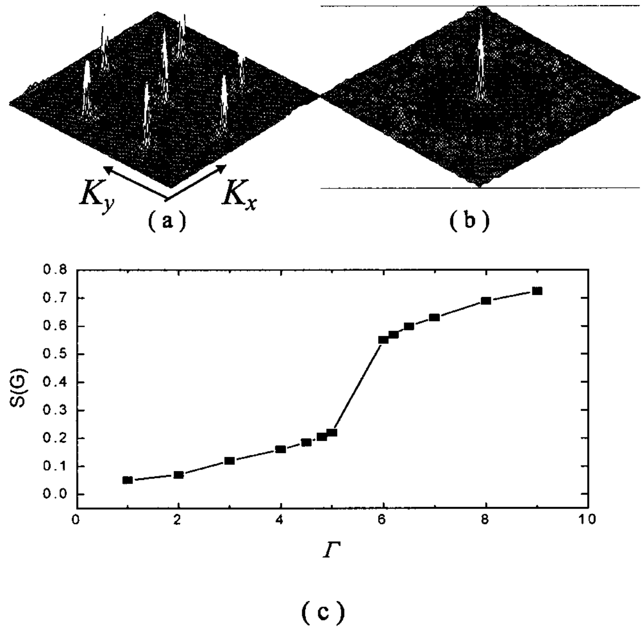

FIG. 2. The structure factor $S(k)$ at depinning for (a) $\Gamma=8$, (b) $\Gamma=1$, (c) averaged intensity of Bragg peaks $S(G)$ vs Γ.

lations on vortex matter [22]. A distinct feature in Fig. 1 is that a substantial decrease occurs in F_c within the crossover from plastic to elastic depinnings. This is also shown in Fig. 3, where we present the F_c versus Γ curve. The corresponding differential curve is plotted in the inset of Fig. 3 where we find a peak.

The strong metastability, e.g., history dependence, illustrates the dominance of disorder [26] and is a strong evidence of plastic flow. Due to the metastability of channels and the defects in moving configurations, the system becomes noisy and history dependent. In the simulations on colloids in this paper, we found a strong history dependence of the depinning process of colloids in the plastic regime, as shown in Fig. 4. Similar behavior appears in vortex simulations with periodic pinning [31]. But no history dependence is found in the elastic regime. A scaling fit between velocity

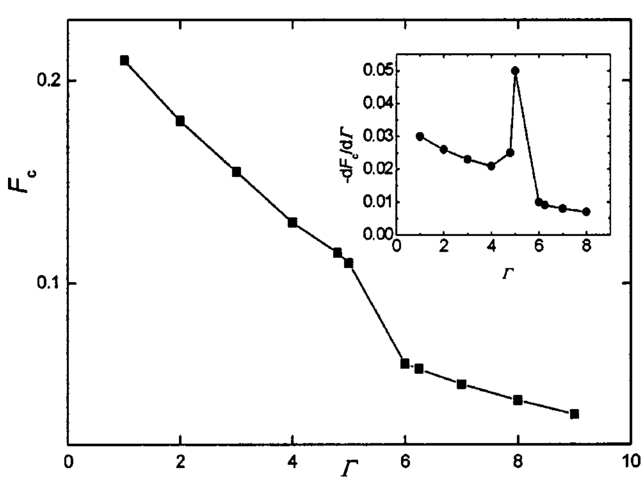

FIG. 3. Depinning force F_c vs Γ. Inset: Corresponding $dF_c/d\Gamma$.

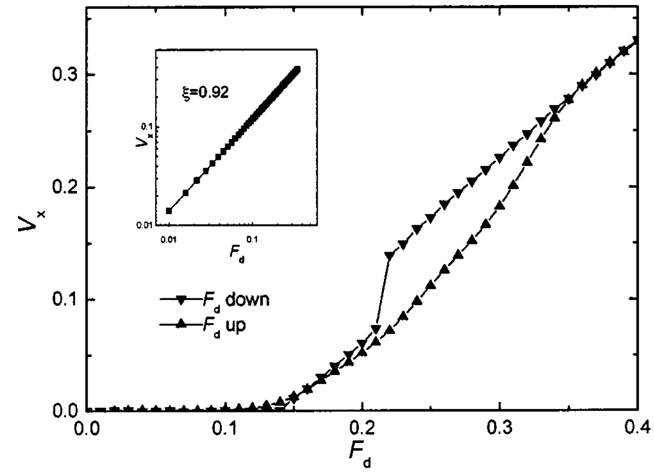

FIG. 4. History depndence of the depinning process in the plastic regime ($\Gamma=3.5$). Inset: Scaling behavior in the elastic regime. The line shows a scaling fit to $(F_d-F_c)^{\zeta}$ with the apparent critical exponent $\zeta=0.92$.

and driving force, i.e., $V_x \sim (F_d-F_c)^{\zeta}$, can be obtained above F_c. This is a general feature of elastic medium. For $\Gamma=8$, the apparent critical exponent is found to be $\zeta=0.92\pm0.01$, as shown in the inset of Fig. 4. This is in agreement with recent experiment ($\zeta<1.0$ is found in colloid experiment [19] on elastic depinning). However, no scaling fit can be obtained in the plastic regime, consistent with simulations on vortex matter [22].

We now investigate the temperature dependence of VFD curves at a fixed value of $\Gamma=0.1$. It is found that F_c decreases with increasing temperature. This is in agreement with the general feature of vortex matter [22]. Furthermore, the VFD curves are strongly influenced by the temperature, as seen in Fig. 5, where we present a set of VFD curves at different temperatures. The log-log plot shows a change in the gross feature of the curves: the VFD curves are approximately linear at high temperatures (above $T^*=0.05\ T_{m0}$). This is the general behavior of liquid. However, at low temperatures

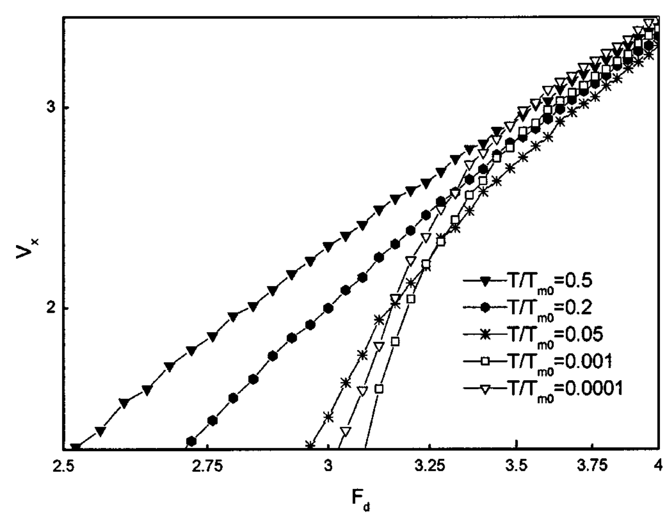

FIG. 5. Series of VFD at different temperatures (the interaction strength between colloids is fixed by $\Gamma=0.1$).

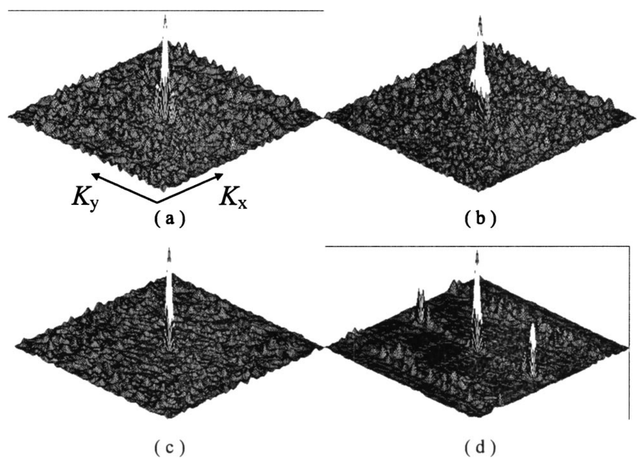

FIG. 6. Structure factor $S(k)$ at different temperatures and different driving forces: (a) $T/T_{m0}=0.5$ and $F_d/F_c=1.1$. (b) $T/T_{m0}=0.5$ and $F_d/F_c=4.0$. (c) $T/T_{m0}=0.001$ and $F_d/F_c=1.1$. (d) $T/T_{m0}=0.001$ and $F_d/F_c=4.0$.

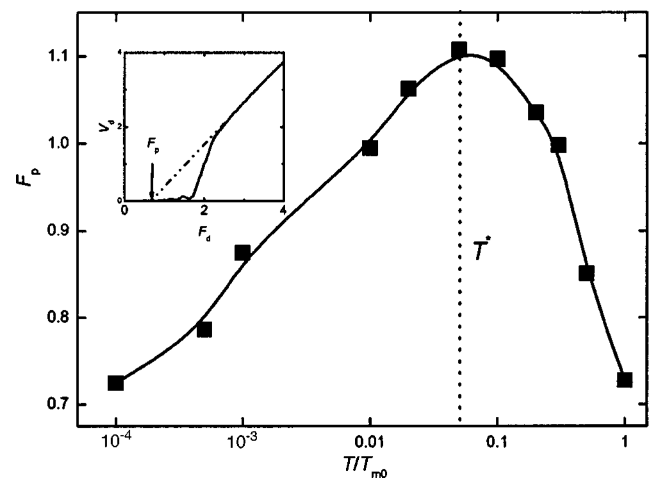

FIG. 7. Dynamical critical driving force vs tmperature. Inset: extrapolation from the linear flow region to obtain the dynamical critical driving force F_p.

(below T^*), the VFD curves turn to a profoundly nonlinear form, consistent with the exponential glass expression [27]. Another obvious feature is that the VFD curves start to cross below T^*, indicating occurrence of an order state [22,28,29]. The idea that an applied driving force can cause colloids to order in the presence of pinning is a simple expression of the fact that an applied driving force tilts the disorder potential, thereby reducing the effective pinning strength. When a large enough force is applied, the colloids depin and flow defectively and then order, and the dynamical friction they experience decreases [28].

To characterize this ordering, we present structure factors $S(k)$ at different temperatures and different driving forces in Fig. 6. From Figs. 6(a) and 6(c), we can see that no order occurs at low driving forces, no matter at low or high temperatures. However, at high driving force, an order takes place as the temperature is decreased below T^*. Comparing Fig. 6(b) with Fig. 6(d), this point can be seen. In Fig. 6(d) we find that Bragg peaks appear only in the $K_x=0$ axis, indicating occurrence of smectic order at low temperatures.

To characterize this ordering further, we reexamine the VFD curves shown in Fig. 5. At high driving forces, all VFD curves exhibit a well-defined linear flow region. If we extrapolate this linearity back to the x axis, as illustrated in the inset of Fig. 7, then the intercept in x axis gives us the dynamical critical driving force F_p [28]. This extrapolated critical driving force is proportional to the average pinning force that the colloids feel in the linear flow region. From Fig. 6 we can find a peak in F_p with decreasing temperature. The peak occurs during the moving liquid to moving smectic transition (near T^*). In the smectic regime, the collective pinning theory of Larkin and Ovchinnikov [30] can give us a qualitative idea of order in the moving configuration. In this theory, the correlation length is given by $R_c \sim \sqrt{1/F_p}$. Therefore, the drop in F_p at low temperatures implies an increase in the correlation area in the linear flow region. From Fig. 6(d) we know that the smectic order begins to appear below T^*.

IV. SUMMARY

To summarize, we have investigated numerically the dynamics of driven 2D colloids subject to the randomly distributed pointlike pinning centers. Decreasing the strength of colloid-colloid interaction, we found a crossover from elastic to plastic depinnings, where a substantial increase in the depinning force was observed. The scaling relationship between velocity and driving force could be obtained above depinning in the elastic regime, and the apparent scaling exponent was found to be $\zeta=0.92$, in agreement with recent experiment on colloids. The influence of temperature was examined, and we found a dynamical phase transition from the moving liquid to the moving smectic by decreasing the temperature. A peak was observed in the dynamical critical driving force across the transition, accompanied by a clear cross of the velocity-force dependence curve.

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